OP10ABE

S4BA · Stat. TIS

Time : 2 ½ hours. Max.Marks : 75

Note : 1) All questions are compulsory
2)Attempt any two sub questions out of four in each question.
3) Graph papers will be provided on request.
4) Calculators are allowed .
5) Figures to the right indicate marks.

Q.1

a) Write the mathematical form of a Linear programming problem . (10)
Explain the following terms for a Linear Programming Problem (L.P.P) :- i) Basic feasible solution and Optimum basic feasible solution
ii) Slack variable and Surplus Variable .

b) A machine is used for producing two products A and B . Product A is produced by using 4 units of chemical salt and 2 units of chemical mixture. Product B is produced by using 2 units of chemical salt and 3 units of chemical mixture . Only 1000 units of chemical salt and 1500 units of chemical mixture are available. The Profit on product A is Rs.30 and on B is Rs. 20 per unit. Formulate this L.P.P (10)

c) Solve the following L.P.P graphically :-
Max $Z = 20x + 30y$
Subject to :- $3x + 3y \leq 36$
$5x + 2y \leq 50$
$2x + 6y \leq 60$
$x \geq 0, y \geq 0$ (10)

d) Solve the following L.P.P using Simplex method upto first iteration table :-
Min $Z = x_1 - 3x_2 + 2x_3$
Subject to :- $3x_1 - x_2 + 3x_3 \leq 7$
$-2x_1 + 4x_2 \leq 12$
$-4x_1 + 3x_2 + 8x_3 \leq 10$
$x_1, x_2, x_3 \geq 0$ (10)

OP10ABE

Q.2

a) Describe the transportation problem .Describe the matrix form of the transportation problem with 3 origins and 4 destinations . Also state the necessary and sufficient condition for the existence of the feasible solution. (10)

b) Determine the initial basic feasible solution to the following transportation problem using :- i) North west corner rule . ii) Matrix minima method. (10)

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | Supply |
|---|---|---|---|---|---|---|
| $O_1$ | 2 | 1 | 3 | 4 | 5 | 38 |
| $O_2$ | 6 | 2 | 8 | 3 | 6 | 52 |
| $O_3$ | 5 | 4 | 3 | 2 | 2 | 75 |
| $O_4$ | 1 | 2 | 4 | 6 | 7 | 15 |
| Demand | 40 | 45 | 50 | 20 | 25 | |

c) Solve the following transportation problem using Vogel's approximation method for minimization of cost. (10)

| Warehouse:- | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ | Supply:- |
|---|---|---|---|---|---|---|
| Factory $F_1$:- | 3 | 4 | 6 | 8 | 9 | 20 |
| $F_2$ | 2 | 10 | 1 | 5 | 8 | 30 |
| $F_3$ | 7 | 11 | 20 | 40 | 3 | 15 |
| $F_4$ | 2 | 1 | 5 | 14 | 16 | 13 |
| Demand :- | 40 | 6 | 8 | 18 | 6 | |

d) A company has three plants and four warehouses .The supply and demand in units and the corresponding transportation costs are given . A table taken from the solution procedure for the transportation problem is given below :- (10)

OP10ABE

| Warehouse | W | X | Y | Z | Supply |
|---|---|---|---|---|---|
| Plant | | | | | |
| A | 7 | 10 | 14 | 8 | 30 |
| B | 7 | 11 | 12 | 16 | 40 |
| C | 5 | 8 | 15 | 9 | 30 |
| Demand | 20 | 20 | 25 | 35 | |

Following is the initial basic feasible solution:-

| Plant :------→ | Warehouses | : SUPPLY |
|---|---|---|
| A :-----→ | X | 10 |
| A : ----→ | Y | 20 |
| B :-----→ | Y | 5 |
| B : ----→ | Z | 35 |
| C : ----→ | W | 20 |
| C : ----→ | X | 10 |

Answer the following questions, giving brief reasons where necessary :-

(i) Is the above solution feasible?

(ii) Is this solution degenerate ?

(iii) Is this solution optimal.

Q.3

a) Explain an Assignment Problem. Give the mathematical formulation of an assignment problem . What is an unbalanced assignment problem ? (10)

b) Solve the following Assignment problem :- (10)

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 15 | 21 | 17 | 4 | 9 |
| B | 3 | 40 | 21 | 10 | 7 |
| C | 9 | 6 | 5 | 8 | 10 |
| D | 14 | 8 | 6 | 9 | 3 |
| E | 21 | 16 | 18 | 7 | 4 |

c) Find the sequence that minimizes the total elapsed time required to complete the following tasks. Also compute the minimum elapsed time. (10)



14

OP10ABE

| Tasks | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| Time on machine_I | 2 | 5 | 4 | 9 | 6 | 8 | 7 |
| Time on machine_II | 6 | 8 | 7 | 4 | 3 | 9 | 3 |

d) Find the sequence that minimizes the total elapsed time in hours required to complete the following jobs on three machines $M_1$, $M_2$, $M_3$ in the order $M_1M_2M_3$. (10)
Also find the total elapsed time.

| Jobs: | P. | Q. | R. | S. | T. |
|---|---|---|---|---|---|
| Machines $M_1$ :- | 5 | 7 | 6 | 9 | 5 |
| $M_2$ :- | 2 | 1 | 4 | 5 | 3 |
| $M_3$ :- | 3 | 7 | 5 | 6 | 7 |

Q.4

a) A patient has been recommended two nutrients $N_1$ and $N_2$ everyday. Minimum intake is 10gm for $N_1$ and 15gm for $N_2$ everyday. These nutrients are available in two products $P_1$ and $P_2$. One unit of $P_1$ contains 2g of $N_1$ and 3g of $N_2$. One unit of $P_2$ contains 1g of $N_1$ and 2g of $N_2$. Cost per unit is Rs.200 for $P_1$ and Rs.150 for $P_2$. (5)
Formulate the above problem as a Linear programming problem.

b) Explain the MODI method of obtaining the optimal solution. (5)

c) ABC company has 4 plants A, B, C and D and each plant can manufacture any one of the four products P, Q, R and S. Production cost and selling price differs from to plant and product to product. The two tables give production costs and selling prices per unit.

Selling price per unit.

| | P | Q | R | S |
|---|---|---|---|---|
| A | 50 | 68 | 49 | 62 |
| B | 60 | 70 | 51 | 74 |
| C | 55 | 67 | 53 | 70 |
| D | 58 | 65 | 54 | 69 |

(5)

OP10ABE

Production cost per unit.

| | P | Q | R | S |
|---|---|---|---|---|
| A | 50 | 68 | 49 | 62 |
| B | 60 | 70 | 51 | 74 |
| C | 55 | 67 | 53 | 70 |
| D | 58 | 65 | 54 | 69 |

Determine which product each plant should manufacture .So as to maximize total profit , given that each plant manufactures one product only.

d ) A business problem is formulated and expressed below as an L.P.P.(Profit is in Rs. and Resources are in units ) (5)

Objective function :- Maximize $Z = 80x_1 + 100x_2$

Subject to resource constraints ,

$$X_1 + 2x_2 \leq 720$$

$$5x_1 + 4x_2 \leq 1800$$

$$3x_1 + x_2 \leq 900$$

$$X_1 \geq 0 , x_2 \geq 0$$

Simplex algorithm of LPP , applied to the above problem yielded following solution.

| | | $C_j$ | 80 | 100 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| B.V | $C_B$ | $X_B$ | $X_1$ | $X_2$ | $S_1$ | $S_2$ | $S_3$ |
| $X_2$ | 100 | 300 | 0 | 1 | $\frac{5}{6}$ | $\frac{-1}{6}$ | 0 |
| $X_1$ | 80 | 120 | 1 | 0 | $\frac{-2}{3}$ | $\frac{1}{3}$ | 0 |
| $S_3$ | 0 | 240 | 0 | 0 | $\frac{7}{6}$ | $\frac{-5}{6}$ | 1 |

(i) Write the Basic feasible solution from the above simplex table.

(ii) Determine whether the solution is optimum , if not find the optimum solution.



१५